Modular Arithmetic

November 4, 2017

Modulo

For two numbers A and B, we say that $A \equiv B \mod 5$ if A and B have the same remainder when divided by 5.

- $8 \equiv 23 \mod 5$ because the remainder of 8 divided by 5 is 3, and the reminder of 23 divided by 5 is also 3.
- $8 \not\equiv 14 \mod 5$ because the remainder of 8 divided by 5 is 3, but the remainder of 14 divided by 5 is 4, NOT 3.

1. (a) Is $13 \equiv 6 \mod 5$?
   (b) Is $85 \equiv 0 \mod 5$?
   (c) Is $17 \equiv 3 \mod 7$?
   (d) Is $5 \equiv 2 \mod 4$?
   (e) Is $4 \equiv -1 \mod 5$?

1 Clocks and Calendars

1. (a) Suppose it's 4PM and someone wants to meet you in 5 hours. When is your meeting time?
   (b) Suppose it's 10AM and someone wants to meet you in 5 hours. When is your meeting?
   (c) Suppose it's 9PM and you have a meeting in 12 hours. When is your meeting?
   (d) Suppose it's 1PM, and you have a meeting in 19 hours. What time is your meeting?
2. (a) 20 minutes after 5:47 is 6:_____
   (b) 37 minutes after 3:52 is 4:_____
3. October 31, 2017 was a Tuesday. There are 31 days in October and 30 days in November.
   (a) What day of the week will November 17 be?
   (b) What day of the week was October 3?
   (c) What day of the week will December 31 be?

Extra Trick: Doomsday Rule

It turns out that the following dates are always on the same day of the year. We will call that day "Doomsday".

- The last day of February
- 4/4
- 6/6
- 8/8
- 10/10
- 12/12
- 5/9
- 9/5
- 7/11
- 11/7

This year (2017) Doomsday is on a Tuesday. So all of those dates are Tuesdays in 2017.

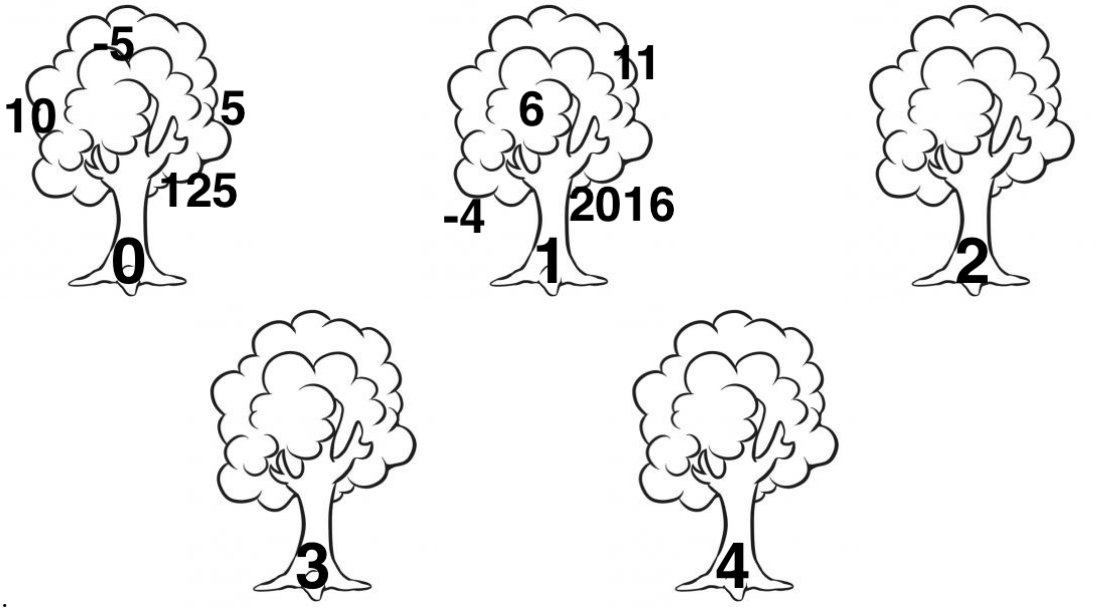
Use the fact that Doomsday is a Tuesday to determine what day the following dates fall on in 2017.

(a) March 7
(b) August 10
(c) September 20
(d) Christmas
(e) Thanksgiving
(f) Your Birthday

2 Mod n Trees

1. Fill in the blanks with the smallest positive numbers possible.
   (a) $76 \equiv$ _____ mod 12
   (b) $52 \equiv$ _____ mod 12
   (c) $76 \equiv$ _____ mod 60
   (d) $15 \equiv$ _____ mod 7
   (e) $15 \equiv$ _____ mod 3
   (f) $15 \equiv$ _____ mod 11

2. Here is a drawing of the world mod 5. On the tree with a 0 on the trunk, we put all the numbers that are congruent to 0 mod 5. On the tree that with a 1 on the trunk, we put all the numbers that are congruent to 1 mod 5. Write at least four numbers on each of the other trees.



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3. For the mod 5 trees above, is it possible to have the same number on two different trees?

4. On another sheet of paper, draw trees for mod 2. How many trees do you need? Draw at least four numbers on each tree. What word could you use to describe the numbers on the 0 tree mod 2? On the 1 tree mod 2?

5. Draw trees for mod 3.

6. Draw trees for mod 4. Is there a relationship between the trees mod 4 and the trees mod 2?