The Number Seven [1]

Warm-up

A six-digit number having 1 as its leftmost digit becomes three times bigger if we take this digit off and put it and the end of the number. What is this number?

[1] Problems from Numberphile and from the Berkeley Math Circle)

1 Dividing by Seven

Below, divide 100 .... 0000 (20 zeroes) by 7.

Do you see a pattern? What is it?

How could you have saved yourself work?

Now, on a piece of scratch paper, divide 20 ... 000 (20 zeroes), 30 ... 000 (20 zeroes), 40 ... 000 (20 zeroes), etc., by 7, and write your results in the table below.

$10\cdots000$ (20 zeros) $\div 7$	
$20\cdots000$ (20 zeros) $\div 7$	
$30\cdots000$ (20 zeros) $\div 7$	
$40\cdots000$ (20 zeros) $\div 7$	
$50\cdots000$ (20 zeros) $\div 7$	
$60\cdots000$ (20 zeros) $\div 7$	
$70\cdots000$ (20 zeros) $\div 7$	

Therefore, the decimal representation of each fraction is as below:

$1 \div 7$	
$2 \div 7$	
$3 \div 7$	
$4 \div 7$	
$5 \div 7$	
$6 \div 7$	
$7 \div 7$	

Now, on a separate sheet of paper, multiply 142857 by 1, 2, 3, 4, 5, 6, and 7, and look at the results.

142857×1	
142857×2	
142857×3	
142857×4	
142857×5	
142857×6	
142857×7	

2 Tricks with 142857

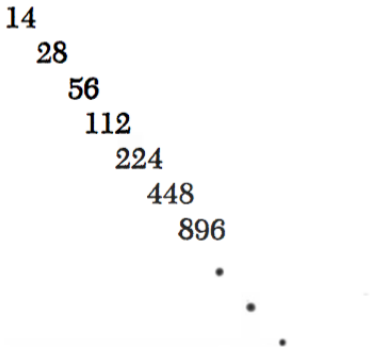
1. Split and add trick.
   (a) Take the number 142857 and split it into three 2-digit by drawing three vertical lines. Add up the three numbers. What do you get?
   (b) Take the number 142857 and split it into two 3-digit numbers by drawing one vertical line in the middle. Add up the two 3-digit numbers. What do you get?
   (c) Take the number 142857 and write it twice. Split the resulting 12 digit number into 4-digit numbers. Add the 4-digit numbers. What do you get?
   (d) Can you find a similar trick adding 5-digit numbers?
2. Multiplication trick.
   (a) Pick any number.N between 1 and 100.
   (b) Multiply 142857 by that number.
   (c) If your answer has more than 6 digits, draw a line to the left of the 6 rightmost digits.
   (d) Add together the numbers on the left and right of the line.
   (e) Compare your answer to your neighbor's answer.

   Try to predict which numbers N give you similar answers and which give you different answers.
3. Addition trick: Consider the following numbers, all permuted from each other.

   142857
   428571
   285714
   857142
   571428
   714285

   Pick any two of the numbers on the list and add them up. Do you get anything interesting?

4. Doubling trick: Start with the number 14, double it to get 28, double that to get 56, and so on. Write each of these numbers in a long list, shifting over by 2 decimal places each time, as below.



Add up this list of numbers. What do you get?

3 Numbers other than Seven

Try some of the following division problems on scratch paper, and make up your own. What patterns do you notice? How can you predict whether the sequence of digits will terminate or repeat?

$10\cdots000$ (20 zeros) $\div 2$	
$10\cdots000$ (20 zeros) $\div 3$	
$10\cdots000$ (20 zeros) $\div 4$	
$10\cdots000$ (20 zeros) $\div 5$	
$10\cdots000$ (20 zeros) $\div 6$	
$10\cdots000$ (20 zeros) $\div 8$	
$10\cdots000$ (20 zeros) $\div 9$	
$10\cdots000$ (20 zeros) $\div 11$	
$10\cdots000$ (20 zeros) $\div 13$	
$10\cdots000$ (20 zeros) $\div 17$	
$10\cdots000$ (20 zeros) $\div$ (you choose)	
$10\cdots000$ (20 zeros) $\div$ (you choose)	