Linear Explorations

Investigation 1: Exploring Linear Functions of the form $ax + by = c$, where $c = a + b$.

Graph the following functions:

$$x + y = 2$$

$$4x - y = 3$$

Create 3 more linear functions of this form, and write them below. Graph them in Desmos (desmos.com) on the same graph as the above functions.

You can use Desmos to explore the graphs of a family of such functions by creating a objects in Desmos called a Slider and a List. You want to graph lines of the form:

$$ax + by = a + b$$

Start by creating slider for values of a (a slider allows you to set a to a particular numerical value and to easily change it later). Type $a = 1$. This will create a slider for a.

Next create a list of values for b. Type $b = [-5, -4.8, \dots, 5]$. Now b is an entire sequence of 51 numbers called a list.

Now type $ax + by = a + b$.

This will graph all the equations of the form $ax + by = a + b$ using your a value from the slider, and then all the values from -5 to 5 for b, increasing by 0.2 each time. This is a quick way to graph 51 lines that all follow the same pattern. Change the value of a to generate 51 more lines.

Use your graph to make a conjecture about linear functions of this form. Write your conjecture in sentence form.

Verify your conjecture algebraically.

Investigation 2: Exploring Linear Functions of the form $ax + by = c$, where $c = ab$.

Graph the following functions:

$$x + y = 1$$

$$5x - 3y = -15$$

$$8x + 5y = 40$$

Create 3 more linear functions of this form, and write them below.

Write what you would type into Desmos to graph a family of functions that follow this pattern. Again, start by assuming you will set and vary a using your slider, and create a sequence of values for b using a list.

Graph this family of lines. Use your graph to make a conjecture about linear functions of this form. Write your conjecture in sentence form.

Verify your conjecture algebraically.

Investigation 3: Linear Functions of the form $ax + by = c$, with a, b, and c in arithmetic progression.

An *arithmetic progression* is a sequence of numbers with a common difference between consecutive terms.

Ex. 2, 5, 8, 11, 14,… is an arithmetic progression (common difference 3)

5, 3.5, 2, .5, -1, … is an arithmetic progression (common difference -1.5)

2, 2, 2, 2, 2, … is an arithmetic progression (common difference 0)

On a new graph in Desmos, graph the following functions, with a, b, and c in arithmetic progression.

$$x + y = 1$$

$$-x + y = 3$$

$$x + \frac{3}{2}y = 2$$

Create three more linear functions of this form, and write them below.

Graph this family of lines by creating a list. You will need to use your parameter a that you decide on, as well as a variable d that represents the common difference. Use your graph to make a conjecture about linear functions of this form. Write your conjecture in sentence form.

Verify your conjecture algebraically.

Investigation 4: Linear Functions of the form $ax + by = c$, with a, b, and c in geometric progression.

A *geometric progression* is a sequence of numbers with a common ratio between consecutive terms.

Ex. 2, 4, 8, 16,… is an geometric progression (common ratio 2)

24, 12, 6, 3, 1.5, … is an geometric progression (common ratio .5)

1, 1, 1, 1, … is an geometric progression (common ratio 1)

On a new graph in Desmos, graph the following functions, with a, b, and c in geometric progression.

$$x + y = 1$$

$$x + 2y = 4$$

$$2x - 6y = 18$$
(*same as*: $x - 3y = 9$)

Graph this family of lines by creating a list. You will only need the variable r that represents the common ratio. Use your graph to make a conjecture about functions of this form. Write your conjecture in sentence form.

Together, let's verify this conjecture algebraically.

Investigation 5: Exploring Quadratic Functions of the form $ax^2 + by = c$, where c = a + b.

Graph the following functions:

$$x^2 + y = 2$$

$$-4x^2 + y = -3$$

Graph this family of curves by creating a list. Use your graph to make a conjecture about functions of this form. Write your conjecture in sentence form.

Verify your conjecture algebraically.

Further Exploration:

Use Desmos to explore relationships among the coefficients and constants of linear and simple quadratic functions. Formulate an interesting question to explore. For example, what would happen graphically with a set of quadratic equations of the form $ax^2 + by = c$ if a, b, and c are related in some particular way (your choice – be specific)? Complete an analysis similar to what we have done today.

1. Clearly specify your question. What relationship are you exploring? Why did you decide to explore this relationship?
2. Create the graph of the family of equations in Desmos.
3. Write a conjecture. How does your conjecture relate to your graph? Consider how you will verify the conjecture algebraically.
4. Write out your algebraic verification, and explain how your algebraic verification supports your conjecture.
5. Based on your explorations, consider other relationships that you might analyze in the future.