Line Segments[1]
January 14, 2017

Warmup problems

1. Points A, B, and C are marked on a line, where AB = 3 and BC = 5. What are the possible lengths of line segment AC?
2. AB = 1, BC = 2, and CD = 4, where A, B, C, and D are points on a line. What are all the possible solutions for the length of AD?

Problems

3. Point B lies on line segment AC, which is 5 cm long. What is the distance between the midpoints of line segments AB and BC?
4. On a straight road going through a village, there are two log cabins 50 meters apart. Where on the road must a well be built so that the total distance from the cabins to the well is minimum?
   a. What if there are three cabins on the road?
   b. ★ Four cabins?
5. Assume 100 students live in village A and 50 in village B. The villages are 3 km apart. If they want to minimize the total distance of the students' walks to school, where should they build the school?
6. Point B divides line segment AC into two parts, with the ratio of their lengths being 2:1. Point D also divides segment AB into two parts, with the ratio of their lengths being 3:2. What is the ratio AD:DB?
7. Points C, D, and E divide line segment AB into ratios 1:2, 1:3, and 1:4, respectively. What is the ratio CD:DE?
8. Six cars are driving from village A to village B. At some time, the cars are at different points on the road, but we know that the total distance (from village A) traveled by all cars is 75 km. We also know that the total distance left for the cars to get to village B is 45 km. What is the distance between the villages?
9. Alice cuts a 100 yard long piece of string into two pieces and holds onto the two cut ends. Then Becky chooses one of the two pieces of string and cuts that piece into two pieces and holds onto the two cut ends. Each player is awarded a prize proportional to the shorter of the two pieces of string she is holding. If both girls are trying to obtain the biggest prize possible, where on the string should Alice make the first cut?
10. ★ Formulate and solve the previous problem for 3 players.

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[1] Based on Alexander Shen's *Geometry in Problems*, from MSRI's Mathematical Circles Library. The last problem comes from an exam by the Wisconsin Mathematics, Science, and Engineering Talent Search.