Introduction to Euclidean Geometry
December 17, 2016

How can one **prove** a mathematical statement? We need three things. First, we define some terms. We then specify a set of statements, called axioms (or postulates), that we consider true. Third, we make an argument to derive new statements from the existing ones. (The argument, in turn, relies on rules of inference.) Once a statement is proven, it becomes a theorem, and we can use it to prove other statements. This is the deductive system of drawing conclusions.

For centuries, the standard for deductive reasoning was Euclidean geometry, in honor of Euclid of Alexandria (4th to 3rd centuries BCE). In his *Elements*, Euclid used 18 definitions, 5 postulates (or axioms), and 6 common notions (also axioms). In 1899, David Hilbert showed that Euclid's system was incomplete, in the sense that Euclid's theorems contained some hidden assumptions not explicitly stated in the axioms. However, it is important to note that even though there were hidden assumptions in his proofs, every result was correct. For us today, Euclid's axioms shall suffice (and a few other statements we might allow).

Euclid's Axioms
1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. **Parallel postulate**: In a plane, given a line and a point not on it, at most one line parallel to the given line can be drawn through the point.

Problems
1. What is the sum of the interior angles of a triangle? Prove it.
2. What is the sum of the interior angles of a quadrilateral? Prove it.
   Is it also true for non-convex quadrilaterals?
3. What can you say about the base angles of an isosceles triangle? Prove it.
4. What is the area of a triangle? Prove it.
5. What is the area of a right triangle if the hypotenuse is 10 cm and the height is 6 cm?[1] Hint: The answer is **not** 30 cm^2.

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[1] According to V.I. Arnold in *Lectures and Problems: A Gift to Young Mathematicians*, this problem appeared for years in an American standardized test for high school students.