Chapel Hill Math Circle

Intermediate Group

Working Backwards[1]

November 5, 2016

1. You have 25 coins and there is exactly one fake coin which is lighter than the rest (all the other coins have equal weight). Using only a balance scale, what is the minimum number of weighings you need to make to guarantee you can determine the fake coin?

2. Two players take turns at removing 1 to 4 coins from an original pile of 16 coins. The player who removes the last coin wins. Is there a winning strategy for either player?
    a. What if we start with 21 coins? How does that change the strategy?

    b. What if we start with 16 coins and can remove up to 5 coins at a time? How does that change the strategy?

    c. What if we start with n coins and remove 1 to k coins at a time, where $k \leq n$?

3. A game board is made up of 25 squares arranged in a 1 x 25 rectangle. In playing the game, two players, Alice and Bob, alternate moving a marker either 1 or 2 spaces forward, with Alice moving first. The marker is initially in a square at one end of the board and always moves toward the other end. The player who cannot make a move that stays on the board loses. Which of the two players can ensure victory?

    a. What if the rules are changed so that the marker can be moved 1, 2, or 3 spaces forward?

[1] The last problem is from *A Moscow Math Circle* by Sergey Dorichenko.