5 Base 12

1. How can we write numbers in a base greater than 10, like 12? How many symbols do we need for the digits?
2. Convert these numbers to base 12. You can use a $1 \leftarrow 12$ machine.
   (a) 12
   (b) 42
   (c) 58
   (d) 135
   (e) 144
   (f) 2012
3. Count to 100_{12} in base 12.
4. Convert to base 10:
   (a) 45_{12}
   (b) AB_{12}
   (c) $10A_{12}$

Extra Problems

5. Calculate
   (a) $248_{12} + 9A7_{12}$
   (b) $42_{12} \times 55_{12}$
6. Write down the multiplication table in base 12.

6 Aliens

1. You are a xeno-archeologist who has found an elementary school textbook from an ancient alien civilization. Although most of the book is no longer legible, you have found one equation that says: $3 \times 4 = 10$. How many fingers do you think the aliens have on each hand?

2. A spaceship full of hostile aliens is about to land on Earth. The aliens are very fond of Earth donuts, and you have persuaded them to leave you in peace in exchange for one donut for each alien on board. The captain radios down and says: "There are 100 of us total on board, and we would like 24 jelly donuts and 32 chocolate donuts with sprinkles." What number system is the alien using and how many donuts do you need to bring them?

Extra Problems

3. Does there exist a number system where the following equations are true simultaneously?

   (a) $3 + 4 = 10$ and $3 \times 4 = 15$?

   (b) $2 + 3 = 5$ and $2 \times 3 = 11$?

4. A blackboard bears a half-erased calculation exercise:

$$\begin{array}{cccccc} & 2 & 3 & ? & 5 & ? \\ + & 1 & ? & 6 & 4 & 2 \\ \hline & 4 & 2 & 4 & 2 & 3 \end{array}$$

   What number system was used and what are the missing digits?

Many of these problems are from *Mathematics Circles: the Russian Experience* by Fromkin, Genkin, and Itenberg