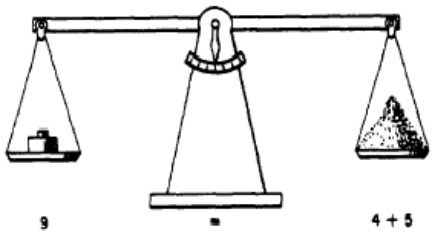
Alien Arithmetic

November 5, 2016

1 Weighing chocolate

1. Using a balance scale, you must be able to balance every whole kilogram amount of chocolate from 1 kg through 15 kg. You may choose four standard weights to use, each a different number of kilograms. Which weights should you use? (Weights may be placed only on the left pan, and chocolate only on the right.)



Extra problems:

2. What is the smallest number of weights you need, so that you can weigh any whole number of grams of chocolate from 1 to 100 on a balance scale? Weights may be placed only on the left pan, and chocolate only on the right.

3. What if you are allowed to place the weights on either pan?

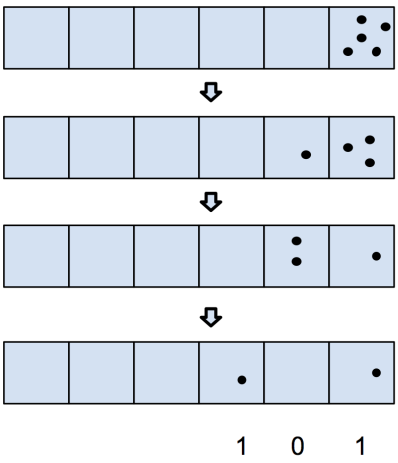
4. How can a chain with 63 links be cut in three places so that you could hand a person any number of links from 1 to 63. (A cut link is still counted as a link.)

2 James Tanton's Exploding Dots

A $1 \leftarrow 2$ machine consists of a row of boxes, extending to the left as far as you'd like. To operate the machine, place a number of dots in the right most box. The machine then redistributes the dots according to the rule:

Two dots in any one box vanish (they explode) and are replaced with one dot one box to their left.

When all the explosions have died down, you can read off a code of 1's and 0's representing the number of dots in each box.



1. What happens if you start with six dots? Eight dots? 25 dots?
2. Now use a $1 \leftarrow 6$ machine. Encode 19. Now encode 42.
3. If your friend came up with the code 152 using a $1 \leftarrow 6$ machine, how many dots did she start with?
4. What is the code for 253 using a $1 \leftarrow 10$ machine?