Four More Advanced Counting Strategies
LEAP Math and MathCounts

1. CLUMPING
   Example: Dr. B, Mr. Mayo, and eight other teachers are standing in line for pizza. If Dr. B and Mr. Mayo insist on standing together so they can chat, how many lineups of all 10 teachers are possible?

   Clump Mayo/Blackburn on one "tile" so there are only 9 tiles to rearrange, and then multiply by 2 to cover all the Mayo/Blackburn arrangements as well as all of the Blackburn/Mayo arrangements.

   9P9 = 9! = 362,880 x 2 = 725,760

2. STICKS AND STONES
   Example: How many ways are there for a grandfather to split $10 among his three grandchildren?

   Picture the ten dollar bills lined up in a row and two sticks that will be used to separate how much each grandchild will get. For example…
   $ $ | $ $ $ $ $ $ $ $ |
   represents the first child getting $2, the second child getting $8, and the third child getting none.

   The problem can then be thought of as choosing the two slots to put the sticks, with 10 + 2 = 12 different slots to place the sticks. Thus, 12C2 = 66 ways to divide the $10.

   Variation: If each child had to get at least $1, then set those $3 aside and the issue is how to divide the remaining $7. Using the same logic, 9C2 = 36 ways.

3. SUB-PROBLEMS
   Example: With 4 girls and 6 boys, how many lineups are possible if all the girls always go first?

   How many ways can the girls be arranged? Each of those arrangements can be paired with each of the different ways the boys can be rearranged. Thus, 4P4 = 24 and 6P6 = 720, so 24 x 720 = 17,280. Note that we never used the fact that there were a total of 10 students.

4. ADDING MULTIPLE PATHS
   Example: What is the probability of winning a "best out of 3" contest if there is only a ¼ chance that you will win any individual match?

   All the paths to victory can be described by WW, WLW, or LWW. Find the prob of each path and then add them together to get the total prob. WW = 1/16, WLW = 3/64, LWW = 3/64. Thus, total prob is 10/64.

1a. 24 b. 11!·9! 2a. 48 b. 480 3a. 12 b. 120 c. 1,814,400

Strategies for Tackling Complex Counting Problems
Smith MathCounts

1. Divide into two sub-problems.
    a. A spinner is spun and can land in one of four equally-spaced colors while a standard die (6 faces) is rolled. How many possible combinations of one color and obe number are possible?

    b. Twenty classmates (9 boys and 11 girls) are going to lineup with all the boys in the back of the line. How many lineups are possible?

2. Clump multiple items that "have to go together" and treat them as one item.
    a. Andrew, Becky, Chuck, Danisha, and Everett are going to lineup. How many possible lineups are there if Andrew and Becky have to stand beside each other?

    b. How many possible schedules might Joey have if he is taking 7 classes that have to be slotted into 7 periods? He is taking PE, French, Band, LA, Math, Social Studies, and Science. Two requirements are that Math and Science have to be in a block of adjacent periods and LA and Social Studies have to be ina block of adjacent periods.

3. Dealing with item repetitions by finding the full number of permutations and then dividing by a number to adjust:
    a. How many distinct four digit numbers could be made using exactly the four digits 1, 2, 2, and 3?

    b. How many distinct six-digit numbers could be made using exactly the six digits 1, 2, 3, 3, 3, 4?

    c. How many distinct permutations of the letters in the word MATHCOUNTS are possible?

Counting Problems (in a Probability Context)
Smith MathCounts Team

1. What is the probability that a random arrangement of the letters in the word STALE makes a real word?

2. How many license plates are possible if each plate is made of three letters followed by four digits?
   a. How many of those license plates would start with ASS?

3. Three five-sided dice are rolled and summed…
   a. Draw a histogram showing how often each sum would be expected to show up…what is the smallest roll, the highest roll, and the most frequent roll? How often would the most frequent roll be rolled (on average)?

   b. If you were betting, would you bet on getting a roll of less than eight or a roll of greater than eleven? Why?

   c. What is the probability that all three dies are greater than three?

4. In a classroom of heterosexuals with 12 boys and 12 girls, how many different dating couples are possible?

5. If 20 people in a room all shake each others hand once, how many total handshakes happened?

6. In an 8-team league in which each team plays every other team twice, how many total league games are played during a season?

7. How many even five-digit numbers can be formed using the digits 3, 4, 5, 6, and 7 once each per number?

8. Ordering a pizza…nine toppings to choose from…four are meat toppings and five are vegetarian. If you order a pizza with two toppings…
   a. How many different pizzas might you have ordered?

   b. What is the probability that your pizza has pepperoni on it?

   c. What is the probability that your pizza is vegetarian?

9. Five classmates (Andrew, Bill, Chuck, Debra, and Evelyn) are asked to stand in line.
   a. How many different orders are possible?

   b. What is the probability that the two girls are at the front of the line?

9c. $\frac{1}{15}$ 10a. $\frac{1}{1140}$ b. $\frac{20}{1140}$ c. $\frac{7}{95}$ 11a. 90 b. $\frac{1}{6}$ c. $\frac{1}{15}$ d. $\frac{2}{3}$

12a. 6840 b. $\frac{1056}{6840}$ c. $\frac{1656}{6840}$ d. $\frac{1}{22}$ 13a. .008 b. .192 c. .808 d. $\frac{1}{2,118,760}$

c. Although Bill and Debra are dating, assume that students took their places in line randomly. What is the probability that Bill and Debra were lucky enough to stand together in line?

10. Marbles in a bag…6 red, 3 blue, and 11 white. A cluster of three marbles are pulled randomly from the bag…
    a. What is the probability that all three marbles are blue?

    b. What is the probability that all three marbles are red?

    c. What is the probability that none of the marbles are white?

11. Getting dressed in the morning, you have to choose from among three pairs of shoes, five pairs of pants, and six shirts…
    a. How many different outfits might you come to school in?

    b. What is the probability that you're wearing the red shirt?

    c. What is the probability that you're wearing the red short and the white shoes?

    d. What is the probability that you're not wearing the green pants nor the purple shirt?

12. The teacher randomly picked three students to be the President, Secretary, and Treasurer of a new class club. If there are 8 boys and 12 girls in the class…
    a. How many different ways might the teacher have assigned the students to the three positions?

    b. What is the probability that a boy is President and girls are both Secretary and Treasurer?

    c. What is the probability that the same gender got all three positions?

    d. Given that all three positions were filled by girls, what are the chances that both Sarah and Julia got a position?

13. If I randomly pick five of the 50 states, what is the probability that…
    a. Hawaii and Alaska are both picked?

    b. Hawaii or Alaska is picked?

    c. Neither Hawaii nor Alaska is picked?

    d. All five states touch the Pacific Ocean?

1. $\frac{6}{120}$ 2. $26^3 \cdot 10^4$ a. 10,000 3. smallest = 3, largest = 15, "9" → $\frac{19}{125}$ 4. $28 = \frac{35}{125}$, $>11 = \frac{20}{125}$ c. $\frac{8}{125}$

4. 144
5. 190
6. 56
7. 48
8. a. 36
   b. $\frac{2}{9}$
   c. $\frac{5}{18}$
9. a. 120 b. $\frac{1}{10}$