Counting[1]
February 25, 2017

Warmup problem

1. In a certain small country there are 4 cities: A, B, C, and D. There are 5 roads between A and B, 4 roads between B and C, 2 roads between A and D, and 3 roads between D and C. In how many different ways is it possible to travel (a) from A to C via B, (b) from A to C via B or D?

Problems

2. In a given nation, every 20th mathematician is also a musician, while every 30th musician is also a mathematician. Are there more mathematicians or musicians? How many times more?

3. There are three bulbs in a very short string of Christmas lights; one is red, another is blue, and the third is green. Each bulb can be either on or off.
   a. In how many different ways can the string light up?
   b. What if the string is made up of five different bulbs of five different colors?

4. An anagram of a word or phrase is a rearrangement of the letters to form a different word or phrase. In this problem, you mean by anagram any permutation of the letters in a word (even if an arrangement of letters is not a word).
   a. How many anagrams does the word REALSPY have?
      i. Can you find one that means an herb?
      ii. Can you find one that describes the members of a team?
      iii. Can you find any others that are English words?
   b. Decipher the following sentence where the correct words are replaced by their anagrams: VOLES ATTACHELAMMI BERMSLOP AYLID.
   c. How many anagrams does the word apple have?
   d. How many does BAOBAB have?

5. Into how many parts can two distinct lines divide a plane? Draw an example of every possible case.
   a. What about three lines?
   b. What about four lines?
   c. ✸ What about n lines?

6. ✸ Is it possible to put beans on the squares of an 8 x 8 grid so there are the same number of beans in any two columns and a different number of beans in any two rows?

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[1] Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.