Chapel Hill Math Circle

Intermediate Group

More Taste of Moscow[1]

February 18, 2017

Warmup problem

1. Which is greater, the sum of all even numbers from 0 to 100 or the sum of all the odd numbers from 1 to 99? By how much?

Problems

2. There are 10 baskets arranged in a circle.
    a. Is it possible to arrange several oranges in the baskets so that the difference in the number of oranges in any two adjacent baskets will be 1?
    b. What if there are three baskets?
    c. What if there are nine baskets?
3. "Andy has more than 1000 books." "No, he has less than 1000 books." "Well, he must have at least one book." If only one of these statements is true, how many books can Andy have?
4. Out of nine identical looking coins, one is counterfeit and lighter than the others. Can you discover the fake coin with only two weighings on a two-pan balance? How?
    a. What is the minimum number of weighings needed to discover a fake coin among 27 coins?
5. The director of a private detective company has put together a list of mutual surveillance assignments for his seven agents, codenamed 001 through 007. Agent 001 will watch the agent who is watching 002; Agent 002 will watch the agent who watches 003; and so on. Agent 007 will watch the agent watching 001. Who watches whom?
    a. Can we do a similar assignment with 8 agents?
6. ★ In a herd of 101 cows, each weighs a whole number of pounds. If any cow is removed from the herd, the remaining cows can be divided into two groups of 50 cows each with the total weight of all the cows in the first group equal to the total weight of all the cows in the second group. Prove that all of the cows weigh the same.
7. ★ Each of the letters F, I, V, and E in this multiplication stands for a different digit:

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    F I V E
  x F I V E
   * * * * F
  * * * * I
 * * * * V
* * * * E
* * * * * * * *
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What are the values of the letters?

[1] Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.