The Dash Instructions. 3 points will be awarded for the first correct answer to each question; 2 points will be awarded for the second correct answer to each question; 1 point will be awarded for the third correct answer to each question. Write your team name, the problem number, and your answer (in simplest form) on your slip of paper before leaving your seat. You may not work on questions while waiting to show your answer to the judges. Teams are allowed to resubmit an answer if their first answer is correct, but must get back in line to do so. You may bring only one solution slip at a time. Good luck, and have fun!

PLEASE DO NOT TURN OVER THIS PAPER UNTIL IT IS TIME TO BEGIN!

1. Solve this cryptogram. Each different letter represents a different digit (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). The same letter always represents the same digit.

$$\begin{array}{r} \text{S U P} \\ +\,\text{S P U} \\ \hline \text{U P S} \end{array}$$

2. What is the greatest common divisor of 312 and 180?

3. What is the greatest common divisor of 451 and 287?

4. Evan is decorating cookies. There are 3 different shapes of cookies (circle, square, and heart), 2 kinds of icing (white and pink), and 4 kinds of sprinkles (chocolate, colored dots, red glitter, and green glitter). How many different types of decorated cookie can Evan make?

5. How many different four-digit numbers can be written using the digits 1, 2, 3, 4, if each digit can be used only once?

6. How many different four-digit **even** numbers can be written using the digits 1, 2, 3, 4, if each digit can be used only once?

7. How many different four-digit **even** numbers can be written using the digits 1, 2, 3, 4 if each digit can be used any number of times?

8. How many positive factors of 36 are also multiples of 4?

9. Find the smallest positive whole number that has a remainder of 4 when divided by 6 and a remainder of 3 when divided by 5.

10. You toss a coin 5 times. How many different sequences of heads and tails can you get? For example, one sequence is HHTHT. Another is THHHT.

11. Each of two card collectors has 20 different Pokemon cards and 10 different Yugioh cards. We call an exchange fair if they exchange a Pokemon card for a Pokemon card or a Yugioh card for a Yugioh card. How many ways are there to carry out one fair exchange between these two collectors?

12. How many 4-digit numbers with the last two digits 38 are divisible by 9? What are the numbers?

13. How many 4-digit numbers with the 2 middle digits 97 are divisible by 45? What are the numbers?

14. How many ways are there to choose a vowel and a consonant from the word ARTICHOKE?

15. How many ways are there to sew a three-colored flag with three horizontal stripes, all the same height but all different colors, if you have six colors of fabric (red, white, blue, green, yellow, black)? We can distinguish the top of the flag from the bottom.

16. Solve this cryptogram. Each different letter represents a different digit (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). The same letter always represents the same digit.

$$\begin{array}{r} \text{A T} \\ \text{A T} \\ +\quad \text{A T} \\ \hline \text{B A T} \end{array}$$

17. How many ways are there to arrange the letters in the word WINTER?

18. How ma ways are there to arrange the letters in the word SUMMER?

19. There are 7 kittens at the animal shelter. Jill is allowed to pick 3 of them to bring home. How many different ways can she do this?

20. How many integers between 1000 and 2000 have all three of the numbers 15, 20, and 25 as factors?

21. How many factors does the number 84 have that are NOT prime? Remember, the number 1 is not prime.

22. You roll a 6-sided die three times. Among all possible outcomes, how many outcomes have NO occurrences of the number 6? For example, 3-6-1 has an occurrence of the number 6, but 3-4-2 does not. Assume the order of the rolls matters here, so 3-6-1 is different from 3-1-6

23. You roll a 6-sided die three times. Among all possible outcomes, how many outcomes have ONE OR MORE occurrence of the number 6? For example, 3-6-1 has an occurrence of the number 6, but 3-4-2 does not. Assume the order of the rolls matters here, so 3-6-1 is different from 3-1-6

24. You have three bags of fruit: a bag of bananas, a bag of pears, and a bag of mangos. How many ways are there to choose one piece of fruit for breakfast, one piece for lunch, and one piece for dinner if you don't want to have the same kind of fruit with all three meals? (The same kind of fruit at two meals out of three is ok.)

25. How many ways are there to pick 1 or more math books to take on vacation if you have a shelf of 5 books to choose from?

26. Solve this cryptogram. Each different letter represents a different digit (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). The same letter always represents the same digit.

$$\begin{array}{cccc} & B & B \\ & & A \\ + & & A \\ \hline & C & C & C \end{array}$$

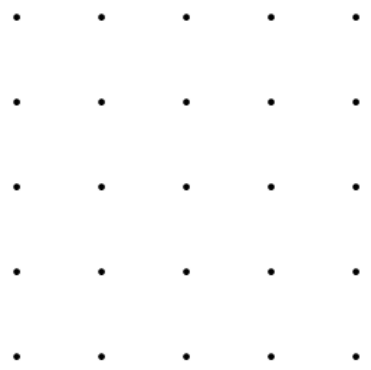
27. The number 10101 is written in base 2. What is its representation in ordinary base 10?

28. The number 1000 is written in ordinary base 10. How is it written once it's converted to base 7?

29. Add $11121_3 + 120110_3$ (in base 3). Give your answer in base 3.

30. Multiply 102_3 by 201_3 (in base 3). Give your answer in base 3.

31. How many different squares can you draw on the lattice below whose vertices all lie on lattice points and whose sides are all horizontal or vertical?



32. How many different squares can you draw on the lattice below whose vertices all lie on lattice points, if the squares are allowed to be tilted?

33. For how many different numbers can you find a square whose *area* is that number, if the square has to be drawn on the lattice below with each vertex on a lattice point?