Note: Developed for the Chapel Hill Math Circle - Advanced HS Group by Boyd Blackburn in March, 2017

General Problem Solving Strategies from James Tanton

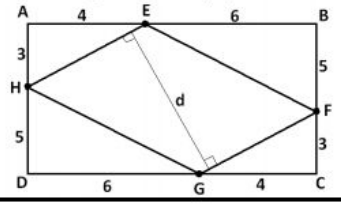
Taken from his Curriculum Inspirations Series sponsored by the MAA

- #1 Engage in Successful Flailing
- #2 Do Something
- #3 Engage in Wishful Thinking
- #4 Draw a Picture
- #5 Solve a Smaller Version of the Same Problem
- #6 Eliminate Incorrect Choices
- #7 Perseverance is Key
- #8 Second-Guess the Author
- #9 Avoid Hard Work
- #10 Go to Extremes

Sample problems…reflect on the general problem solving strategies you are using (in additional to your specific math content knowledge).

1.

In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



2.

What is the correct ordering of the three numbers 10^8, 5^{12} and 2^{24}?

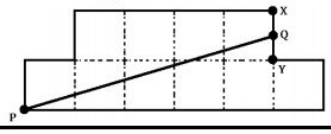
(A) $2^{24} < 10^8 < 5^{12}$
(B) $2^{24} < 5^{12} < 10^8$
(C) $5^{12} < 2^{24} < 10^8$
(D) $10^8 < 5^{12} < 2^{24}$
(E) $10^8 < 2^{24} < 5^{12}$

3.

Suppose that $|x+y|+|x-y|=2$. What is the maximum value of x^2-6x+y^2?

4.

The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?



5.

Consider all quadrilaterals $ABCD$ such that $AB=14$, $BC=9$, $CD=7$, and $DA=12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

6.

At a party, there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men?

7.

The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

8.

A set of 25 square blocks is arranged into a 5 x 5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

9.

Every day at school, Jo climbs a flight of 6 stairs.
Jo can take stairs 1, 2 or 3 at a time. For example, Jo could climb 3, then 1, then 2 stairs.
In how many ways can Jo climb the stairs?

10.

In the United States, coins have the following thicknesses:

penny, 1.55 mm;
nickel, 1.95 mm;
dime, 1.35 mm;
quarter, 1.75 mm.

If a stack of these coins is exactly 14 mm high, how many coins are in the stack?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

11.

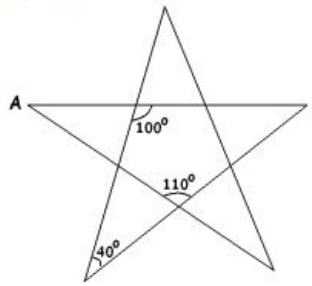
Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are 1,3, 4, 5, 6, and 9. What is the sum of the possible values for w?

12.

Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called *n-ray partitional* if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

13.

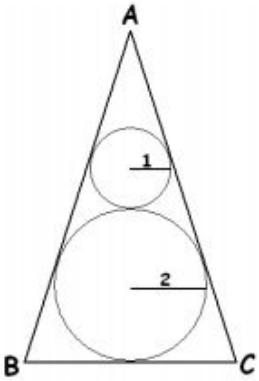
The degree measure of angle A is ...?



14.

This month's challenge is Question 3 from the AMC 10/12 Practice Quiz on "Geometry of Triangles, II."

A circle of radius 1 ***is tangent to a circle of radius*** 2***. The sides of*** ΔABC ***are tangent to the circles as shown, and the sides*** $\overline{AB}$ ***and*** $\overline{AC}$ ***are congruent. What is the area of*** ΔABC***?***



15.

In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

16.

A majority of the 30 students in Ms. Demeanor's class bought pencils at the bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was $\$17.71$. What was the cost of a pencil in cents?

17.

How many positive two-digit integers are factors of $2^{24}-1$?

18.

What is the units digit of $19^{19}+99^{99}$?

19.

There are 52 people in a room. What is the largest value of n *such that the statement "At least* n *people in this room have birthdays falling in the same month" is always true?*

20.

In a room, $\frac{2}{5}$ of all the people are wearing gloves, and $\frac{3}{4}$ of the people are wearing hats. What is the minimal number of people in the room wearing both hats and gloves?

Answer Key (follow weblinks for worked out solutions)

1. 7.6
2. A
3. 8
4. 2/3
5. 2sqrt(6)
6. 3/8
7. 64
8. 600
9. 24
10. 8
11. 31
12. 2320
13. 30
14. 16sqrt(2)
15. 224
16. 11
17. 12
18. 8
19. 5
20. 20