Counting Week 1 Warm-Up

- *Chapel Hill Math Circle*

1. Venn Diagrams
    - Use a Venn Diagram to represent the idea that at Chapel Hill High, the girls' Fall volleyball team has 14 players and their Winter basketball team has 12 players. Four of the girls play on both teams.
    - How would you modify your diagram to explicitly include the 650 girls at the high school that do not play on either team?

2. Tree Diagrams to illustrate the Fundamental Counting Principle
    - Use a tree diagram to represent how many different one-topping pizzas you could order if you had a choice of thick or thin crust, and pepperoni, mushrooms, sausage, and onions for the toppings. How many different pizzas are possible?
    - How would you modify your diagram if someone could choose two toppings?

3. Organized Lists
    - Make an organized list of all the possible orderings ("permutations") of the letters in the word "PART." How many orderings are possible?
    - If you added one letter and made it "PARTS", how many permutations would be possible? What is the pattern that would allow you to predict how many such permutations would be possible with 10 distinct letters?
    - We can easily turn these issues into probability questions...Back to "PART", what is the probability that a permutation of PART forms a real English word?

BONUS - TRY THESE AT HOME

Counting Quiz (Batterson Unit) v2 **Name** _____

- *LEAP Math 2*

_____ 1. What is the probability that two randomly chosen people were born on the same day of the week?

_____ 2. How many three-digit numbers can be formed using only eights and nines?

_____ 3. If you are in a class of 32 people and the teacher randomly selects three of you to pass out papers, what is the probability that she picks the only three students with red hair?

_____ 4. I am looking at (in no particular order) a yellow, red, green, and blue car parked in a row. What is the probability that the red and green car are parked next to each other?

_____ 5. If you can order a pizza with any two of eight different toppings, how many different orders (that include two distinct toppings) are possible?

_____ 6. How many different arrangements of the letters in the word "TWEET" are possible?

_____ 7. If 20 students come to school on a cold day and 15 are wearing gloves and 12 are wearing hats, how many students are wearing both a glove and a hat if exactly two students foolishly came to school with neither a glove nor a hat?

_____ 8. If three 8-sided dice (1 - 8 for each face) were rolled and summed, how many possible sums are there?

_____ 9. In a grid of city blocks that is three blocks tall and six blocks wide, Jerome is standing in the bottom right corner. He needs to walk to work which is in the top left corner. He has to walk on the sidewalks and cannot cut diagonally across any blocks. Over a year, he takes lots of different paths to work that all count as the "shortest path." How many different "shortest paths" are possible?

_____ 10. 7 girls and 3 boys stand in a line. How many different line orders are possible if the girls always stand in the front of the line and the boys always stand in the back?

D. Venn Diagrams

1. In a survey, students were asked to check boxes indicating whether they would like to have a cat as a pet or a dog as a pet, and they were allowed to check both boxes. Ten students checked "cat," 15 students checked "dog," and 3 students didn't check either box. If 24 students were surveyed, how many students would like both a cat and a dog? [abb]

2. In a class of 30 students, there are 8 Asian students, 3 of whom play soccer. If there are 9 soccer players in the classroom, what is the probability that a student picked at random is neither Asian nor a soccer player? [abb]

3. A group of baseball players were surveyed about their chewing gum tastes. It was found that:
   - 22 liked juicy fruit
   - 25 liked spearmint
   - 39 liked bubble gum
   - 9 liked both spearmint and juicy fruit
   - 17 liked juicy fruit and bubble gum
   - 20 liked spearmint and bubble gum
   - 6 liked all three
   - 4 liked none of these

   What percentage of the players liked all three flavors? [hrci]

E. Intro to Counting/Combinatorics

1. There are how many arrangements of the five letters in the word LEAST? How many of them spell a word? [abb]

2. Five siblings were playing in the living room when they broke a lamp. Nobody wants to have to be the ones to tell Mom and Dad, so they have agreed to draw straws, and the two that get the shortest straws will go together to tell their parents. How many pairings of siblings are possible? [abb]

3. How many three-digit numbers can be formed using the digits 1, 2, 3, or 4 with each digit being used once at the most? How does your answer change if you allow the digits to be repeated?

F. License Plates and Phone Numbers

1. Within Charlotte's 704 area code, how many 7-digit phone numbers are possible given that a phone number cannot start with 0 or 1? How many would be possible if a digit could only be used once? [abb]

2. Nebraska, the home of the AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by four digits. Each new license plate consists of three letters followed by three digits. By how many times is the number of possible license plates increased? [amc8]

3. Tiles with the digits 1 – 6 are placed in a bag. You form a three-digit number out of this set in the following way: pull a tile and place it in the hundreds position, pull a second tile and place it in the tens position, and pull a third tile and place it in the units position. How many three-digit numbers are possible? What is the probability that a number is made of only odd digits? [abb]

G1. 42
G2. 12
G3. 168
G4. 2160 / 3/7

H1. 720
H2. 720
H3. 6
H4. 6

I1. 6, 15, 20, 15, 6, 1
I2. 15
I3. 126
I4. 56
I5. 756

J1. 28
J2. 120
J3. 4

G. Factorials and Canceling

1. $\frac{7!}{5!}$ 2. $\frac{5!}{10}$ 3. $\frac{4! \cdot 7!}{6!}$

4. How many arrangements of all the letters in the word CAREFUL start with a vowel? What is the probability that a rearrangement starts with a vowel?[abb]

H. Permutations

1. There are 10 horses in a race. Gamblers can bet on something called the *trifecta*. In order to win, you must correctly guess which horse finishes 1st, 2nd, and 3rd (in the correct order. How many possibilities for the trifecta are there in a race with 10 horses? [agm]

2. How many ways can five books be ordered on a shelf from left to right? [agm]

3. Using only the odd digits {1,3,5,7}, how many two-digit numbers are possible in which the units digit is larger than the tens digit? [abb]

4. How many distinct four-digit numbers can be formed by rearranging the four digits in 2010? [amc]

I. Combinations

1. Starting with a group of six friends:
   - How many one-person groups are possible?
   - How many two-person groups are possible?
   - Three-person groups?
   - Four-person groups?
   - Five-person groups?
   - Six-person groups?

2. There are six Scrabble letters left in the bag at the end of the game (F,H,J,S,U, and Y) and you are going to pick two. How many different pairs of letters might you end up with? [agm]

3. Selecting from the nine-element set D = {1, 2, 3, 4, 5, 6, 7, 8, 9}, how many four-element subsets can be made? [hrci]

4. How many of these four-element subsets include the digit 5? [hrci]

5. How many four-digit numbers have four different digits? [hrci]

J. Picking vs Not Picking Equivalence

1. You are painting a masterpiece and you're going to use every one of your 8 paint colors except for two. How many color combinations might you select? [abb]

2. You have 10 shirts to choose from as you pack for a week-long trip. Assuming you take one shirt for each day, how many combinations of shirts might you end up with? [abb]

3. Ms. Hamilton's eight-grade class wants to participate in the annual three-person basketball tournament. Lance, Sally, Joy, and Fred are chosen for the team. In how many ways can three starters be chosen? [amc]

D1. 4
D2. 16/30
D3 12%

E1. 120, 5 (6 if you include "TEALS"
E2. 10
E3. 24, 64

F1. 8,000,000 / $\overline{483,840}$
F2. 67.6
F3. 120 / 1/20