Chapel Hill Math Circle

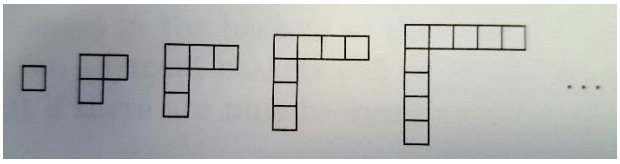
Intermediate Group

A Taste of Moscow[1]

February 11, 2017

Warmup problems

1. Which is greater, $333333 \cdot 444444$ or $222222 \cdot 666667$? By how much?
2. Here's a series of figures:



How many squares are in the 100th figure? How many squares are in the first 100 figures altogether?

Problems

3. A grasshopper jumping along a straight line can jump 6 or 8 inches in either direction. Can it reach a point that is:
   a. 1.5 inches away from its original position;
   b. 7 inches away;
   c. 4 inches away?
4. A cardboard rectangle with area 1 is cut into two pieces along a line segment that connects the midpoints of two adjacent sides. Find the areas of the two pieces.
5. The windows on a subway train are as shown in the drawing. The curves forming the rounded corners are all arcs of a circle. A portion of the window is open 10 inches. The height of the open section is 25 inches. What is the area of the opening?

6. ★ Suppose in a given collection of 2002 integers, the sum of any 100 of them is positive. Prove that the sum of all 2002 of the integers is positive.
7. An ant is sitting in a corner of the floor of a cubical room. It wants to move to the opposite corner using the shortest route. It can only move along the walls, floor, and ceiling of the room. What path should it take?
8. ★ A bus, a truck, and a motorcycle pass a stationary observer at equal time intervals. They pass another observer farther down the road at the same equal time intervals, but in a different order. This time the order is bus, motorcycle, truck. What is the speed of the bus, if the speed of the truck is 30 mph and the speed of the motorcycle is 60 mph?
9. ★ The numbers 2^{2002} and 5^{2002} are expanded and their digits are written out consecutively on one page. How many total digits are on the page?

[1] Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.