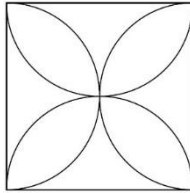Chapel Hill Math Circle
Intermediate Group (6-7)
March 12, 2016

1.  Warm-up: In Fairyland, every day is either a sunny day or a rainy day. Moreover, every sunny day is immediately preceded by two consecutive rainy days. Also, five days after a rainy day, there is another rainy day. Today is a sunny day. For how many days, at most, can we accurately predict the weather?

2.  How many natural numbers less than 2016 have an odd number of positive divisors?

3.  Thirty scientists attend a conference. Each one is either a mathematician, a physicist, a chemist, or a biologist. The combined number of physicists and biologists is half the number of mathematicians. The combined number of physicists and chemists is twice the number of biologists. There is at least one physicist. How many mathematicians are there?

4.  If we draw four semicircles on each side of a square with its sides as diameters, we obtain a four-petal flower. The area of the square is 64 cm$^2$. What is the area of the four combined petals?



5.  Prove that if 51 numbers are chosen from the integers between 1 and 100, inclusively, then at least 2 of them must be consecutive.