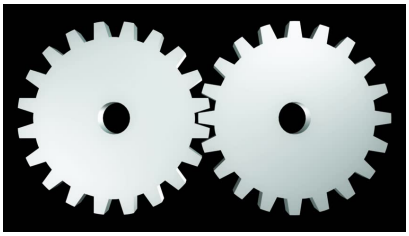
Two is Better than Infinity - Part 3

1 Warm-Up

1. Max placed 11 interlocking gears in a row. Each gear, except for the first and last, is connected to its right and left neighbors. Max turns the leftmost gear clockwise. Is the rightmost gear going to rotate clockwise or counterclockwise?



2. (From last time) A beaver is cutting several long logs into smaller pieces. The beaver makes 30 cuts and ends up with 36 pieces of wood. Each cut goes through one log. How many long logs did he start with?

3. (From last time) You are cutting a round loaf of bread with a knife . What is the maximum number of pieces of bread that you can get if you are allowed to make only three straight cuts? Each cut should be a through cut, and you cannot stack or otherwise rearrange pieces between cuts.

2 Problems

4. Bella placed her 11 interlocking gears in a circle, in such a way that every gear is connected to two neighbors. She turns the first gear clockwise. What happens then?

5. A chess rook stands at the bottom left corner of a 5 x 5 chess board – the one marked with a star in the figure. Being lazy, this rook does not like to move far – in a single step it moves only one square up, down, right, or left. For exercise, the rook is supposed to tour the entire chessboard, visiting each square just once.

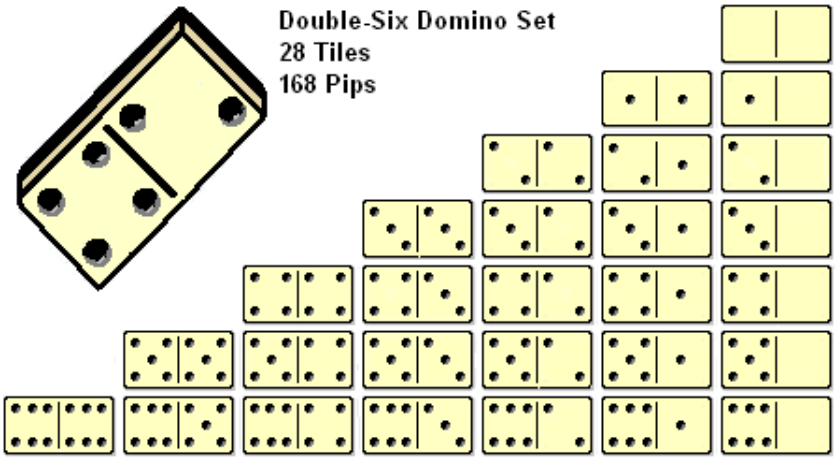
   (a) Can the rook end its walk on the square marked with a flower?

   (b) Can the rook end its walk on the square marked with a smiley face?

   (c) Can the rook end its walk back on the square with the star?

6. (From last time) Suppose you place a full set of dominos (all 28 pieces) into a line according to the rule of dominos (see below), and you make sure that the left half of the leftmost domino has 6 spots on it. How many spots will there on the right half of the rightmost domino?

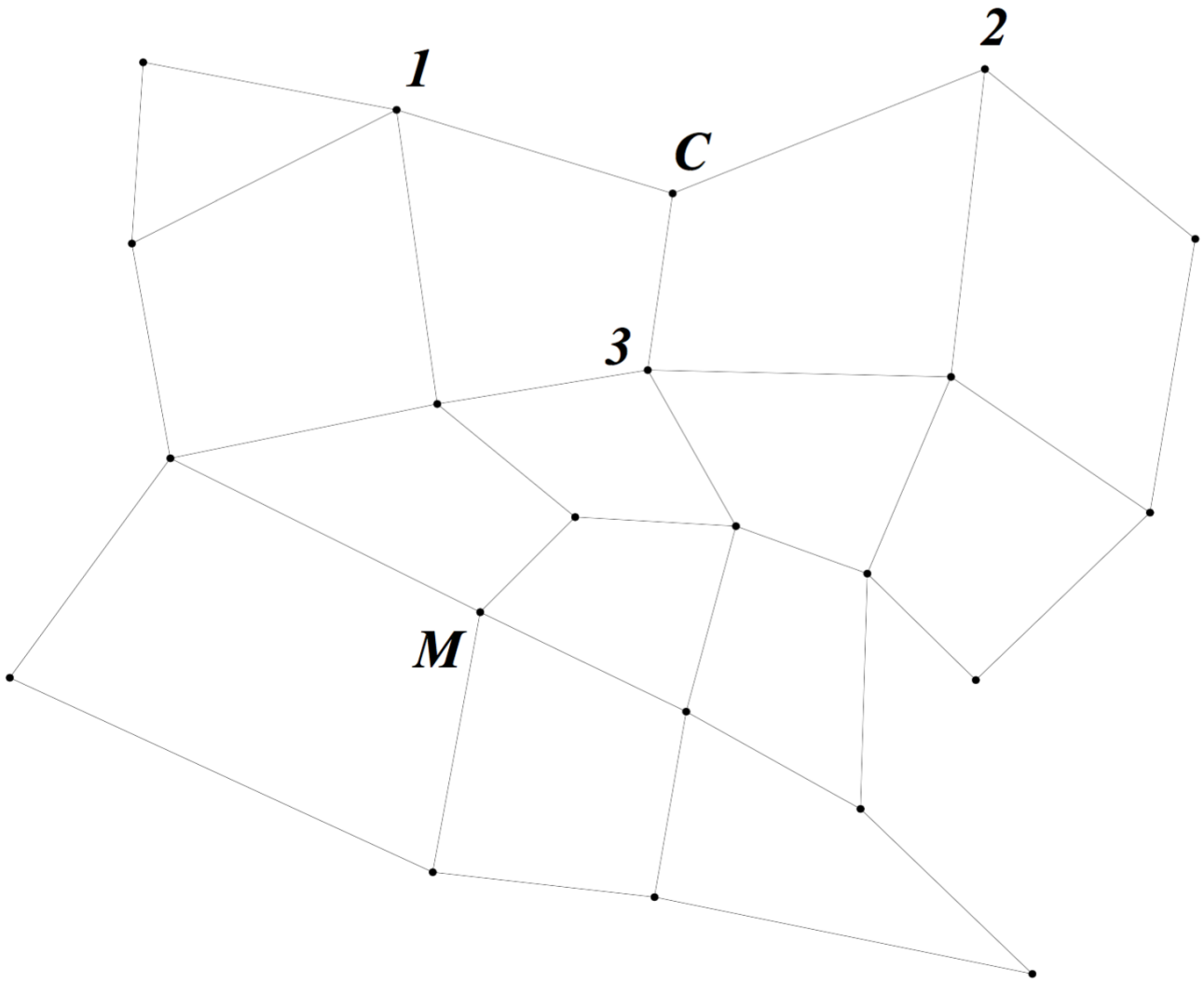
   Rule of Dominos: When two dominos are placed next to each other, end to end, the halves that are next to each other must have the same number of spots.



This week's problems are from *Mathematical Circle Diaries, Year 1* by Anna Burago, from *Solve This!* by James Tanton, and from Paul Zeitz.

3 A Cat and Mouse Game

7. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?