Two is Better than Infinity

1   Warm-Up

There are three towns in a county: A, B, and C. The residents of town A never lie, those from town B never tell the truth, and those from town C alternate true and false statements.

One day, the county fire station received a call: "There is a fire in our town!" When the firefighter asked where the fire was, he got the reply: "In town B." Which town should the firefighters go to?

2   A Game with Cups

With a partner, arrange 7 cups to look like this, with four cups turned upside-down and three cups turned right-side-up.

You are allowed to turn cups over two at a time.

- Two upside-down cups can both be flipped over and placed back on the table right-side-up.

- Two right-side-up cups can both be flipped over and placed back on the table upside-down.

- A pair with one right-side-up and one upside-down cup can be flipped so that they are upside-down and right-side up, respectively.

It is against the rules to flip just one cup or more than two cups at the same time.
You win the game if you can bring all 7 cups to the upside-down position.

3   Problems

1. How can you cut a round pizza with three straight cuts into 4 parts? 5 parts? 6 parts? 7 parts? Cuts must go all the way through the pizza, starting and ending at the outer edge, and you can't rearrange pieces between cuts.

This week's problems are from *Mathematical Circle Diaries, Year 1* by Anna Burago.

2. Louis has reprogrammed the elevator in the 30 story Wayside School. Only two buttons work and the first button sends the elevator 8 floors up and the second one 6 floors down. (The elevator will not move if it is asked to go above the 30th floor or below the 1st floor.)

   (a) Principal Kidswatter is drinking coffee on the first floor. Can he take the elevator to the 23 floor? If so, show how. If not, explain why.

   (b) Can he take the elevator to the 24 floor? If so, show how. If not, explain why.

   *Wayside School experts, please ignore the fact that there is no 19th floor.

3. Three types of magic fruit - apples of wisdom, pears of bravery, and plums of kindness – all grow on the Magic Tree in the Queen's garden. If fruits are picked, they immediately grow back according to the following rules:

   - If a single fruit is picked from the tree, another of the same kind grows in its place.
   - If 2 apples are picked, 4 pears grow back.
   - If 2 pears are picked, 4 plums grow back.
   - If 2 plums are picked, 4 apples grow back.
   - If 2 fruits of different kinds are picked, nothing else happens.
   - It is not possible to pick more than 2 pieces of fruit at a time.

   Currently the tree has 11 apples, 10 pears, and 8 plums. The wicked witch plans to steal all the fruit, by sneaking into the garden several nights in a row and picking one or two pieces of fruit at a time.

   Is there a way for her to pick all the fruit on the tree? Explain how or explain why not.

   Could she pick all but one plum? Explain how or explain why not.

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4   Extra Problems

4. A beaver is cutting several long logs into smaller pieces. The beaver makes 30 cuts and ends up with 36 pieces of wood. How many long logs did he start with?

5. Kolya placed a full set of dominos (all 28 pieces) into a line according to the rule of dominos (see below). The left half of the leftmost domino has 6 spots on it. How many spots are there on the right half of the rightmost domino?

   Rule of Dominos: When two dominos are placed next to each other, end to end, the halves that are next to each other must have the same number of spots.

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