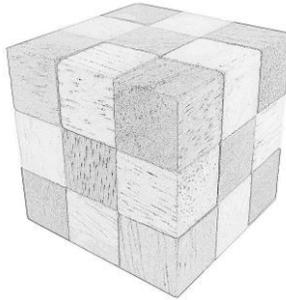Chapel Hill Math Circle
Intermediate Group (6-7)
February 27, 2016

No one shall expel us from the Paradise that Cantor has created.[1] – David Hilbert

1. Hungry the Termite has identified a Rubik's cube made of the most succulent woods. She wants to taste every single unit cube in the Rubik's cube. She plans to start at the center of any side of the Rubik's cube and bore a path that will take her once through every unit cube, without returning to taste a wood already sampled. For reasons only known to her, her path must always be parallel to two sets of opposite faces and perpendicular to the third set of faces of the Rubik's cube. Can Hungry the Termite taste every wood in such a way that the last cube she tastes is the one in the center of the cube (because it's the most humid)? If she can do it, show a way; if she cannot, explain why.

2. A family of five members must cross a perilous bridge at night over some rapids. To do this, they must cross the bridge at most two at a time, otherwise the bridge may collapse. Moreover, they only have one flashlight that they must take back and forth so they can see where they step. It takes 1 minute for Alice to cross the bridge, 3 minutes for Bob, 6 minutes for Momma Cleo, 8 minutes for Papa Dave, and 12 minutes for Grandpa Earl. In addition, they estimate that the flashlight's batteries will last 30 minutes. Can they all safely cross the bridge?

3. Imagine a hotel with infinitely many rooms along an infinitely long hallway. You are the hotel manager and all rooms are occupied. During a thunderstorm, a person walks in at night and requests a room. You are concerned about the safety of this guest and wish to provide him with a room. How can you do this given that all rooms are taken and you cannot force clients to share their rooms? What if instead of one person, a bus arrives with infinitely many people. What can you do now?

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[1] *Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können.*