Chapel Hill Math Circle
Intermediate Group (6-7)
February 20, 2016

1. A logician and a mathematician were arguing. The logician tells the mathematician: "I lied to you yesterday." The mathematician replies, "I also lied to you yesterday." One of them lies only on Mondays, Tuesday, and Wednesdays, whereas the other lies only on Thursdays, Fridays, and Saturdays. When did they talk?

2. Consider the following die. The sides that you cannot see are all prime numbers. Opposite faces always add up to the same. What number is opposite to 14?


3. At a large school, there are 1000 lockers, all on one wall of a long corridor. The lockers are numbered, in order, 1, 2, 3, … ,1000, and to start, each locker is closed. There are also 1000 students, also numbered 1, 2, 3, … , 1000. The students walk the length of the corridor, opening and closing lockers according to the following rules:

   Student 1 opens every locker.
   Student 2 closes every second locker; that is, lockers 2, 4, 6, 8, …, 1000.
   Student 3 changes the state of every third locker, closing it if it is open and opening it if it is closed.
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   Student n changes the state of every $n^{th}$ locker, etc.

   When all 1000 students have walked the corridor, which lockers end up open?

4. Hungry the Termite has identified a Rubik's cube made of the most succulent woods. She wants to taste every single unit cube in the Rubik's cube. She plans to start at the center of any side of the Rubik's cube and bore a path that will take her once through every unit cube, without returning to taste a wood already sampled. For reasons only known to her, her path must always be parallel to two sets of opposite faces and perpendicular to the third set of faces of the Rubik's cube. Can Hungry the Termite taste every wood in such a way that the last cube she tastes is the one in the center of the cube (because it's the most humid)? If she can do it, show a way; if she cannot, explain why.